# Measuring Rotation of the Milky Way from HI Emission

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## Abstract

We observe the 21 cm hyperfine transition of galactic hydrogen using a radio telescope, over a 24 hour period. Using signal processing equipment, spectra from the plane of the galaxy in the range 1418.5-1421 MHz are constructed. These are used to study the rotation of the Milky Way galaxy, and are compared with the predictions of the flat rotation curve model.


## 1 Introduction

Our galaxy contains a great deal of atomic hydrogen spread out fairly smoothly around the plane of the galaxy. One of the most distinctive features of the radio spectrum of the galaxy is the HI emission line at 1420.379 GHz ($\lambda = 21$ cm). Since different parts of the galaxy are moving towards and away from the Sun at different velocities, the Doppler shifts in the HI emission line may be used to discern the radial velocities of HI sources in different parts of the galaxy.

Using the velocities of the atomic hydrogen continuum, a complete model of galactic rotation may be constructed. This assumes, of course, that stars in the galaxy are moving at similar velocities to the surrounding interstellar hydrogen.

## 2 Methods

### 2.1 Radio Telescope Setup

We used the 4 m parabolic radio telescope on the roof of Space Sciences Building at Cornell, set at an elevation of 32.8°. Given that Ithaca, NY is at a latitude of 42.4°N, the declination of the dish was set to 9.6°. Therefore, the line of sight of the dish would be oriented in the galactic plane at right ascension 06:31:40 ($b = 0$, $l = 202.13°$), and again at RA 19:11:24 ($b = 0$, $l = 43.73°$)[1]. Thus we expected the two galactic plane crossings in a given sidereal day to occur approximately 12:39:44 apart (about 760 minutes).

Our radio telescope operates in a passive manner known as a drift scan, sweeping across a band of the sky as the Earth rotates. We observed for a period of three consecutive solar days, starting at 1:25PM EST on November 15, 2002.

[1] I used E. Murphy's equatorial to galactic coordinate converter, set for epoch 2002, available with source code on the Internet: `http://fuse.pha.jhu.edu/support/tools/eqtogal.html`

### 2.2 Signal Processing

From the telescope, the radio signal was sent to a receiver that accepts frequencies in the range 1400-1440 MHz. The receiver used heterodyning to mix the signal down to baseband, making it easier to analyze digitally. The heterodyning process involves mixing the signal with a series of local oscillators, each step producing a lower-frequency signal from the "beats" of constructive interference between the radio signal and the local oscillator.

The first local oscillator was at a frequency of 1159.983 MHz (measured value) in our setup. Mixed with a galactic HI signal at $\sim$ 1420 MHz, this produced a new signal at around 260 MHz (= 1420 MHz $-$ 1160 MHz). This shifted signal was again mixed with a second local oscillator at 212.770 MHz (our measured value) and finally a third LO at 47.000 MHz.

Summing the frequencies of the three LO's and subtracting them from the rest frequency of the HI line, the final analog signal was centered around 626 kHz. The signal was then baseband mixed and digitized at a sampling rate of 10 kHz. A hardware board then Fast Fourier Transformed a 2.5 MHz band (corresponding to 1418.5-1421 MHz from the original signal).

### 2.3 Post-Processing

The FFT spectra were post-processed with a PC by time-averaging them to obtain 2 min time resolution, then rescaled to the proper frequency range around 1420.379 MHz using the measured LO frequencies. Additionally, system temperature variations (due to time of day/night) were corrected.

Finally, we extracted spectra for three regions of particular interest from the first 24 hours of observation:

1. We summed the spectra from 82 minutes to 186 minutes (relative to start of observation), during which period the telescope line of sight passed through the plane of the galaxy in the first quadrant. (spectrum 1 on chart)

2. We summed from 267 to 348 minutes, during which the telescope passed out of the plane of the galaxy and then into the second quadrant. (spectrum 2 on chart)

3. We summed from 743 to 859 minutes, during which the telescope passed through the plane of the galaxy again in the first quadrant. (spectrum 3 on chart)

It would have been helpful to compare these three spectra with spectra for the same lines of sight from the second and third days of observing. This would have helped to reduce measurement errors. However, I was unable to access the complete raw data (to produce more spectra) in time for this report.

## 3 Results and Discussion

### 3.1 Qualitative description

A plot of the continuous spectra from the fast Fourier transform (with 5 min time resolution) is attached. As can be seen, the signal is periodic every 1400 seconds or so, that is once per sidereal day.

As can be seen from this plot, there is a dark broad band at around 130 min, corresponding to the galactic plane crossing in the first quadrant. At about 800 min, the telescope line of sight crossed the galactic plane in the third quadrant, and there is another dark band in the FFT spectrum.

Then, 760 minutes later, the telescope crossed the plane in the first quadrant again, as expected (see 2.1).

Note also that, overall, the HI line appear to oscillate sinusoidally between 1420.25 MHz and 1420.5 MHz. This significant perturbation turns out to be due to the rotation of the Earth, and we must correct for it, since in order to build a model of the rotation of atomic hydrogen in the galaxy, we must understand its motion relative to the Sun.

## 3.2 Correcting for the Earth's motion

We wish to use the Doppler shift to measure the velocities of HI sources in the galaxy with respect to the Sun. In order to do this, we must correct for the interaction of Earth's orbital motion around the Sun with its own daily rotation. This correction varies over the course of a day as the component of Earth's velocity along the line of sight changes.

The semi-major axis of the Earth's orbit is $1.50\times10^{11}$ m, and thus the circumference of its orbit is approximately $9.42\times10^{11}$ m. Given an orbital period of approximately 365.25 days ($3.16\times10^{7}$ s), the velocity of the Earth in its orbit is $\sim 2.98\times10^{4}\ \mathrm{m\,s^{-1}}$. From the Doppler effect, we expect a periodic shift in the HI line due to the orbit of the Earth: when the line of sight is along the direction in which the Earth is moving, the HI line will be blue-shifted (from a source that is stationary with respect to the Sun). The HI line will be red-shifted when the line of sight is opposite the motion of the Earth. Hence the period of this shift will be one sidereal day.

The maximum blue shift due to Earth's rotation will be (using negative $v$, per the convention):

$$\begin{aligned} \frac{f}{f_0} &= \sqrt{\frac{1-v/c}{1+v/c}} = 1 + 9.94\times10^{-5} \\ f &= 1420.379 \text{ MHz} \times 1.0000994 = 1420.520 \text{ MHz} \end{aligned}$$

While the maximum red shift will be (using positive $v$):

$$\begin{aligned} \frac{f}{f_0} &= 1 - 9.94\times10^{-5} \\ f &= 1420.379 \text{ MHz} \times 0.9999006 = 1420.238 \text{ MHz} \end{aligned}$$

We can state that the component of the Earth's velocity along the line of sight as a function of time (positive is away from objects viewed) is:

$$v_{\oplus} = (-2.98\times10^{4}\ \mathrm{m\,s^{-1}}) \cos\left[\frac{2\pi(t-t_0)}{1436 \text{ min}}\right]$$

The length of a sidereal day is approximately 1436 min. The first point of maximal blue-shift (most negative $v_{\oplus}$) is seen from the attached FFT graph to occur at around 1200 min, so $t_0 \approx 1200$ min.

The predictions of the above formula agree very well with our FFT graph: a faint HI band oscillates sinusoidally between around 1420.25 MHz to 1420.5 Mhz, with a period of around 1430 minutes. This band corresponds to the HI emissions of sources that are approximately at rest with respect to the Sun.

## 3.3 Plane crossings

### 3.3.1 First quadrant crossing

The first plane crossing visible on our FFT chart is at about 150 min, and corresponds to the plane crossing at RA 19:11:24, Galactic latitude 43.73°. This is in the first quadrant. The spectrum is attached and marked as (1). The midpoint of the time period over which this spectrum is integrated is at 134 min, and at this time $v_\oplus \approx 1.43\times10^3\ \mathrm{m\,s^{-1}}$.

The HI spectrum for this period appears very "bumpy," and it is somewhat difficult to delimit the HI line. It is fairly clear that the broad tall peak between 1420.15 MHz and 1420.4 MHz is due to HI emission, but what about the much smaller separate peak at around 1420.65 MHz? This peak occurs in neither of the other two spectra plotted on the same chart. Apparently, this peak belongs to the HI spectrum as well, and the bumpiness of the spectrum is due to that fact that as the telescope looks in towards the galactic center in the first quadrant, atomic hydrogen is unevenly distributed in the intervening spiral arms of the galaxy.

I bounded the HI "line" using the outermost pair of intensity peaks (local maxima), which occur at around 1420.12 Mhz and 1420.65 MHz. Then I took the outer half-maxima of these two peaks, relative to the adjacent continuum, and found these to occur at 1420.04 MHz and 1420.70 MHz. Essentially, I treated the whole thing as one large tilted plateau and tried to take the full-width half-maximum, recognizing that the "maximum" value was different at each side of the peak.

Using these bounds for the HI line, I computed that the atomic hydrogen along the line of sight has radial velocities:

$$\frac{V_r + v_\oplus}{c} = \frac{1-(f/f_0)^2}{1+(f/f_0)^2} \qquad \text{(rearranging Doppler shift formula)}$$

$$-6.77\times10^4\ \mathrm{m\,s^{-1}} \lesssim V_r + v_\oplus \lesssim 7.16\times10^4\ \mathrm{m\,s^{-1}}$$

Here $V_r > 0$ indicates a receding source again. Correcting for the motion of the Earth around the Sun, we get:

$$-6.92\times10^4\ \mathrm{m\,s^{-1}} \lesssim V_r \lesssim 7.01\times10^4\ \mathrm{m\,s^{-1}}$$

As expected, we observe both blue-shifted and red-shifted gas in the first quadrant. The red-shifted gas lies between us and the tangent point of the line of sight, and the blue-shifted gas lies past the tangent point (according to the galactic rotation model outlined in 3.4). In the attached spectrum 1, we see that the intensity (temperature) of the red-shifted gas is greater than that of the blue-shifted gas, which we expect since the red-shifted gas is closer to us and we will therefore measure greater intensity for its radiation.

### 3.3.2 Third quadrant crossing

The second plane crossing visible on our FFT chart is at about 800 minutes, and corresponds to the plane crossing at RA 06:31:40 ($b = 0$, $l = 202.13°$). This is in the third quadrant and the spectrum is attached and marked (3). The midpoint of the integration period is at 801 minutes, at which time $v_\oplus = 5.19\times10^3\ \mathrm{m\,s^{-1}}$.

The HI line in this spectrum is a single intense narrow peak. This time, I computed the width simply as the full-width half-maximum. The half-maxima range was from approximately 1420.285 MHz to 1420.432 MHz. Thus the HI sources along this line of sight have radial velocities:

$$\begin{array}{ccccc} -1.12\times10^4\ \mathrm{m\,s^{-1}} & \lesssim & V_r + v_\oplus & \lesssim & 1.98\times10^4\ \mathrm{m\,s^{-1}} \\ -1.64\times10^4\ \mathrm{m\,s^{-1}} & \lesssim & V_r & \lesssim & 1.47\times10^4\ \mathrm{m\,s^{-1}} \end{array}$$

These values are anomalous! They do not agree with the galactic rotation model outlined in 3.4 below. Since all the hydrogen gas in the third quadrant is at least as far from the galactic center as the Sun is (follows directly from the geometry of galactic coordinates), the minimum $V_r$ in the third quadrant ought to be zero! Yet we have found negative $V_r$ (blue-shift) from our spectrum.

Therefore I am very distrustful of these values for the radial velocities in the third quadrant. I am not sure why they are so incorrect. Perhaps I have not found the correct boundaries for the HI line in spectrum 3, although since it is a single fairly symmetric peak, the FWHM should be sufficient.

### 3.4 Comparison with galaxy rotation model

The Doppler shift of a cloud of hydrogen gas measures the radial velocity of that cloud with respect to the Sun:

$$\begin{aligned} V_r &= \left\| \vec{V} - \vec{V_0} \right\| \\ &= R_\odot \left[ \frac{V(R)}{R} - \frac{V(R_\odot)}{R_\odot} \right] \sin l \quad , \end{aligned}$$

where $R$ is the distance between the gas being studied and the center of the galaxy, and $V(R)$ is the linear velocity at that point (assuming circular symmetry for the rotation). $R_\odot$ represents the position of the Sun. If the galaxy has a roughly flat rotation curve as observed, $V(R) = V_0 =$ const., and so:

$$V_r = V_0 \left[ \frac{R_\odot}{R} - 1 \right] \sin l \qquad (*)$$

#### 3.4.1 Measurement of $V_0$ and error analysis

It is shown in the handouts that in the first quadrant, the tangent point (the point on the line of sight closest to the galactic center) will have radial velocity:

$$V_t = V_r(R_t) = V(R_t)\,\mathrm{sgn}(l) - V(R_\odot)\sin(l)$$

But if the galaxy has a flat rotation curve:

$$V_t = V_0[\mathrm{sgn}(l) - \sin(l)]$$

From the starred equation defining $V_r$ above, it can be seen that $V_t$ is the maximum value of $V_r$ for a given line of sight in the galactic plane.

Since we have produced a spectrum from the first quadrant of the plane of the galaxy, with known longitude $l$, we can use the maximum velocity observed at this longitude to deduce $V_0$ for our galaxy: for our spectrum (1), the line of sight is $l = 43.73^\circ$, and the maximum velocity, determined from the Doppler shift, is $7.01 \times 10^4\ \mathrm{m\,s^{-1}}$. Thus:

$$\begin{aligned} V_0 &= V_t/[\mathrm{sgn}(l) - \sin(l)] \\ &= (7.01 \times 10^4\ \mathrm{m\,s^{-1}})/[1 - 0.691] \\ &= 2.27 \times 10^5\ \mathrm{m\,s^{-1}} \end{aligned}$$

The accepted value of $V_0$ is $\sim 2.20 \times 10^5\ \mathrm{m\,s^{-1}}$, so my value is too high by 3.21%. This is a fairly large error, but it probably comes from a *tiny* error in measuring the Doppler-shifted HI frequencies. In order to have calculated the canonical value for $V_0$, I would have had to measure the maximum velocity along the line of sight as $6.79 \times 10^4\ \mathrm{m\,s^{-1}}$, with respect to the Sun. Since the maximum velocity is determined from the Doppler shift, this would correspond to a maximum red-shift of

$f_{min}/f_0 = 1 - 2.91 \times 10^{-4}$, or $f_{min} = 1420.05$ MHz. The value I in fact measured from our data was $f_{min} = 1420.04$ MHz.

Thus a very small error of only 10 kHz in measuring $f_{min}$ translates into a substantially larger error in $V_0$! Combining the above formula for $V_0$ with the formula for the maximum radial velocity in the first quadrant, I get:

$$\begin{aligned} V_0 &= \frac{c\left[\frac{1-(f_{min}/f_0)^2}{1+(f_{min}/f_0)^2}\right] - v_\oplus}{1-\sin(l)} \\ &\approx \frac{\frac{c}{2}\left(\frac{f_0^2 - f_{min}^2}{f_0^2}\right) - v_\oplus}{1-\sin(l)} \qquad \text{approximating } 1 + (f_{min}/f_0)^2 \approx 2 \\ &\approx \frac{\frac{c}{2}\left(\frac{2f_0(f_0 - f_{min})}{f_0^2}\right) - v_\oplus}{1-\sin(l)} \qquad \text{approximating } f_{min} + f_0 \approx 2f_0 \\ &= \frac{c\,(1 - f_{min}/f_0) - v_\oplus}{1-\sin(l)} \end{aligned}$$

An incorrect value for $v_\oplus$ could have produced the error in $V_0$. This is quite plausible. In determining $v_\oplus$, I had to determine the time offset (that is $t_0$) by eyeballing the FFT spectrum, because I did not have access to the raw data. Moreover, I used a simplified model of the Earth's rotation around the Sun, which assumed that the Earth's orbit is circular.

Another possible explanation for why my value of $V_0$ is wrong is that my value of $l = 43.73°$ is incorrect. If I assume my values of $V_t$ and $v_\oplus$ to be correct, then in order to obtain the canonical value of $V_0$, the true value of $l$ would have to be $42.95°$. I used an online coordinate converter program to determine the values of $l$ at which the telescope would pass through the plane of the galaxy, set for epoch 2002, and checked the values obtained several times. It is possible that my value of $l$ is off by nearly $0.78°$, since spectrum (1) was averaged over a period of 104 minutes, and so the beam of the telescope obviously did not lie exactly in the plane of the galaxy for this whole time period.

There are at least three reasons why my value for $V_0$ may be off by 3.21%:

1. I did not measure the maximum red-shift along the line of sight quite accurately enough. Perhaps my half-maximum method of locating the edge of the HI line is not the best way to do it.

2. I did not take into account the velocity of the Earth along the line of sight ($v_\oplus$) with sufficient accuracy. Looking at the raw FFT data to determine the exact time offset would help improve this, as would taking into account the slight eccentricity of the Earth's orbit around the Sun, and adjusting the velocity according to the predictions of Kepler's Laws.

3. The spectrum for quadrant 1 is averaged over a 104 minute period during which the Earth passed through the plane of the galaxy. I assume that it passed through the plane exactly in the center of this time period. Naturally the HI spectrum for this period will be a little "fuzzy" around the edges because it is not the spectrum at the *exact* moment at which the telescope beam crossed the galactic plane.

#### 3.4.2 Determining the extent of the atomic hydrogen disk

Suppose that galactic atomic hydrogen extends to a finite distance, $R_{max}$, from the galactic center in the plane of the galaxy. Now using the formula $V_r = V_0[R_\odot/R - 1]\sin l$, it is found that the extremum radial velocity along a line of sight is:

$$\begin{aligned} V_{ext} &= V_0\left[\frac{R_\odot}{R_{max}} - 1\right]\sin l \\ \Rightarrow \quad \frac{R_{max}}{R_\odot} &= \left[\frac{V_{ext}}{V_0 \sin l} + 1\right]^{-1} \end{aligned}$$

There is an ambiguity as to whether to choose the *minimum* velocity observed along a given line of sight, or the *maximum* velocity along that line of sight. Consider the starred equation defining $V_r$ above: if $\sin l$ is positive (as in the first quadrant) then maximum $R$ will correspond to negative $V_r$ (greatest blue-shift). If $\sin l$ is negative (as in the third quadrant), then maximum $R$ will correspond to positive $V_r$ (greatest red-shift). Thus for determining the size of the disk from the first quadrant measurements, we use $V_{r,min}$, while for the third quadrant we use $V_{r,max}$.

The data from both quadrant 1 and quadrant 3 spectra may be used to determine $R_{max}$ experimentally. Using our values for $V_{r,min}$ in quadrant 1, $V_{r,max}$ in quadrant 3, $l$, and $V_0$:

$$\begin{aligned} \frac{R_{max}}{R_\odot} &= \left[\frac{-6.92\times10^4 \text{ m s}^{-1}}{(2.27\times10^5 \text{ m s}^{-1})(\sin 43.73°)} + 1\right]^{-1} = 1.79 \qquad \text{(quadrant 1 values)} \\ \frac{R_{max}}{R_\odot} &= \left[\frac{1.47\times10^4 \text{ m s}^{-1}}{(2.27\times10^5 \text{ m s}^{-1})(\sin 202.13°)} + 1\right]^{-1} = 1.21 \qquad \text{(quadrant 3 values)} \end{aligned}$$

These values do not agree very well. $R_\odot$ is estimated to be somewhere around 10 kPc, and the handouts describe atomic hydrogen gas as stretching out to "several tens of kPc from the galactic center." So both of the above values come up short. Perhaps this is because our equipment was not powerful enough to measure faint HI sources from the outer part of the galactic plane. Also, as I mentioned above in 3.3.2, I believe that the measured value of $V_{r,max}$ in the third quadrant is incorrect, though I am unable to explain it satisfactorily.

## Computer program

I wrote a handy Perl script, `HI.pl`, to calculate radial velocities from Doppler shifts and vice versa. It automatically adjusts for the velocity of Earth along the line of sight when appropriate values for $t$ and $t_0$ are entered, and it prints out all of its calculations along the way.

```
#!/usr/bin/perl
use Math::Trig;
$|=1; sub input($) { print "$_[0] "; chomp($_ = <STDIN>); return $_ }

$c = 299_792_458; #m/s
$f0 = 1420.379; #MHz
$t0 = 1200; #min (time of first occurence of maximal blue-shift)
$t = $Vearth = nan; #set to invalid values so user doesn't forget to set them

until ($done) {
```

```
    print "f0 = $f0 MHz ; t0 = $t0 min ; t = $t min => Vearth = $Vearth m/s\n";
    print "(1) set t0 (2) set t (3) f->v (4) v->f (0) quit: ";
    chomp ($choice = <STDIN>);

    if ($choice == 0) {
        $done = 1;
    } elsif ($choice == 1) {
        $t0 = input("What is the time of first maximal blue-shift (in min)?");
    } elsif ($choice == 2) {
        $t = input("What is the time of the observation (in min)?");
        $Vearth = (-2.98e4)*cos(2*pi*($t-$t0)/1436); # vel. of earth along LOS
    } elsif ($choice == 3) {
        $f = input("Frequency (in MHz)?");
        $ffrac = $f/$f0;
        $beta = (1 - $ffrac**2) / (1 + $ffrac**2);
        $v = $beta * $c;
        $Vr = $v - $Vearth;     # adjust for earth motion

        print ("f/f0 = $ffrac = 1 ", ($ffrac<1 ? "-" : "+"), " ",
               abs(1-$ffrac), "\n");
        print "(Vr+Vearth)/c = $beta\n";
        print "Vr+Vearth = $v m/s\n";
        print "*** Vr = $Vr m/s\n";
    } elsif ($choice == 4) {
        $Vr = input("Velocity Vr (in m/s)?");
        $v = $Vr + $Vearth;
        $beta = $v/$c;
        $ffrac = sqrt((1-$beta)/(1+$beta));
        $f = $ffrac * $f0;

        print "Vr+Vearth = $v\n";
        print "(Vr+Vearth)/c = $beta\n";
        print ("f/f0 = $ffrac = 1 ", ($ffrac<1 ? "-" : "+"), " ",
               abs(1-$ffrac), "\n");
        print "*** f = $f MHz\n";
    }
    print "\n";
}
```

## Graphs

1— FFT spectra of the frequency band around the rest frequency of the HI transition, spread over the 3-day observation period, with 5 min time resolution.

2— Three spectra extracted in order to study the rotation of the galaxy: (1) corresponds to $l = 43.73°, b = 0$, (3) corresponds to $l = 202.13°, b = 0$, and (2) is somewhere in between in quadrant 2, but not in the plane of the galaxy.